ÉRETTSÉGI VIZSGA • 2009. május 5.

MATEMATIKA
ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI
ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

OKTATÁSI ÉS KULTURÁLIS
MINISZTÉRIUM

Important Information

Formal requirements:

1. The papers must be assessed in **pen and of different colour** than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
2. The first one among the shaded rectangles next to each question contains the maximal score for that question. The **score** given by the examiner should be entered into the other **rectangle**.
3. **In case of correct solutions**, it is enough to enter the maximal score into the corresponding rectangle.
4. In case of faulty or incomplete solutions, please indicate the corresponding **partial scores** within the body of the paper.

Substantial requirements:

1. In case of some problems there are more than one marking schemes given. However, if you happen to come across with some **solution different** from those outlined here, please check the parts equivalent to those in the solution provided here and do your marking accordingly.
2. The scores in this assessment **can be split further**. Remember, however, that the number of points given for any item can be an integer number only.
3. If the answer is correct and the argument is clearly valid then the maximal score can be given even if the actual solution is **less detailed** than that in this booklet.
4. If there is a **calculation error** or any other flaw in the solution, then the score should be deducted for the actual item only where the error has occured. If the candidate is going on working with the faulty intermediate result and the problem has not suffered substantial damage due to the error, the subsequent partial scores should still be given.
5. If there is a **fatal error** within an item (these are separated by double lines in this booklet), then even formally correct steps should not be given any points, whatsoever. However, if the wrong result obtained by the invalid argument is used correctly throughout the subsequent steps, the candidate should be given the maximal score for the remaing parts, unless the problem has been changed essentially due to the error.
6. If an **additional remark** or a **measuring unit** occurs in brackets in this booklet, the solution is complete even if the candidate does not mention it.
7. If there are more than one correct attempts to solve a problem, it is the **one indicated by the candidate that can be marked**.
8. You should **not give any bonus points** (points beyond the maximal score for a solution or for some part of a solution).
9. You **should not reduce** the score for erroneous calculations or steps unless its results are actually used by the candidate in the course of the solution.
10. **There are only 4 questions to be marked out of the 5 ones in part II. of this exam paper.** Hopefully, the candidate has entered the number of the question not to be marked in the square provided for this. Accordingly, this question should not be assessed even if there is some kind of solution contained in the paper. Should there be any ambiguity about the student's request with respect to the question not to be considered, it is the last one in this problem set, by default, that should not be marked.

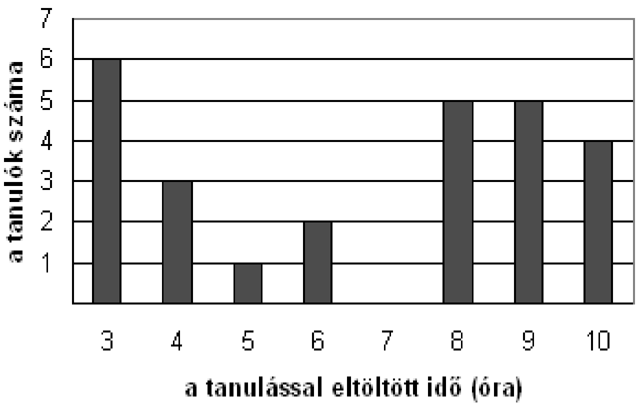
I.

1.

a)

$\bar{x} = \frac{3\cdot 6+4\cdot 3+5\cdot 1+6\cdot 2+7\cdot 0+8\cdot 5+9\cdot 5+10\cdot 4}{26}$.	2 points	
$\bar{x} = \frac{172}{26}$ hours ≈ 6,6 hours.	1 point	*Correct answers without dimensions are worth 1-1 point, each. If the calculation of the median and the mode is not based on the 26 data items then no points can be given for these items.*
Mode: 3 hours.	2 points	
Median: 8 hours.	2 points	
Total:	**7 points**	

b)

	3 points	
Total:	**3 points**	

2.

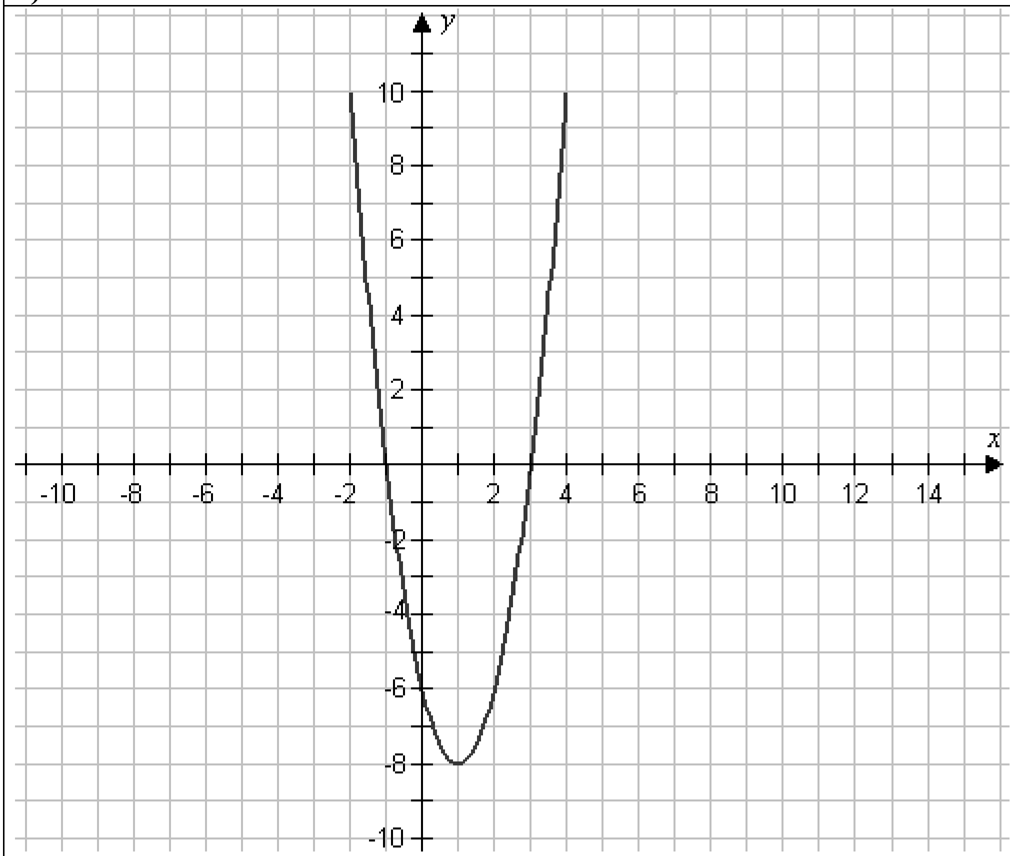
a)

The unit price of type-A is x and that of type B is y.	1 point	
Writing down the conditions: $20x + 30y = 93000$;	2 points	
$30x + 20y = 87000$.	2 points	
The solution of the system is: $x = 1500$ and $y = 2100$.	4 points	*If there is a calculation error then at most 2 points may be given*
The unit price of the respective types is 1500 Ft, and 2100Ft.	1 point	
Total:	**10 points**	

b)

Denote the amount of type A by *a kg*. Then there is $60 - a$ kg of type B is used.	1 point	
Hence $1500a + 2100(60 - a) = 120000$;	1 point	
$a = 10$.	1 point	
10 kg of type A and 50 kg of type B is used for the mixture.	1 point	
Total:	**4 points**	

3.		
a)		
$2x^2 - 4x - 6 = 0$. $x_1 = 3$.	1 point	
$x_2 = -1$.	1 point	
$y = 2 \cdot (x-1)^2 - 8$.	2 points	*These 3 points are due even if the x value yielding the minimum is calculated as the mean of the zeros.*
The value of x yielding the minimum is $x = 1$.	1 point	
The value of the minimum is $y = -8$.	1 point	
Total:	**6 points**	

b)	
	3 points
Total:	**3 points**
If the graph is not on the given interval then at most 2 points may be given.	

c)		
Int he equation of the parabola $a = 2$,	1 point	
therefore $\left(\frac{1}{2p} = 2\right)$; $p = \frac{1}{4}$.	1 point	
The focus is above the vertex at a distance $\frac{p}{2}$,	1 point	
and thus $F\left(1; -\frac{63}{8}\right)$.	1 point	
Total:	**4 points**	

If the nswer for part a) is wrong but the candidate is working correctly in parts b) and / or c) then the corresponding scores are due.

4.		
Finding the domain: I. $x^2 - 3x \ge 0$.	1 point	
$x \le 0$ or	1 point	
$x \ge 3$.	1 point	
II. $x + 2 > 0$.	1 point	
$x > -2$.	1 point	
I and II: $-2 < x \le 0$ or $3 \le x$.	1 point	
The product is negative if its factors are of different signs.	1 point	*The 3 points are also due if the first statement is not formulated.*
Since $\sqrt{x^2 - 3x}$ cannot be negative, hence both $\sqrt{x^2 - 3x} > 0$ and $\log_{0.1}(x+2) < 0$ must be true.	2 points	
The solution of the square-root inequality: $-2 < x < 0$ or $3 < x$.	1 point	*For not allowing the cases of $x = 0$, $x = 3$.*
Since the logarithm function above is strictly decreasing, therefore $x + 2 > 1$.	1 point	*The point is also due for the correct inequality without an explanation.*
$x > -1$.	1 point	
The set of solutions: $-1 < x < 0$ or	1 point	
$3 < x$.	1 point	
Total:	**14 points**	

II.

Out of problems 5 to 9, do not assess the one indicated by the candidate.

5.

Solution 1.

The terms of the geometric progression are a; $b = aq$ and $c = aq^2$.	1 point	
The terms of the first arithmetic progression are a; aq; $aq^2 - a - 2aq$.	1 point	
The terms of the second arithmetic progression are a; $aq + 9$; aq^2.	1 point	
From the first arithmetic progression: $aq = \dfrac{a + aq^2 - a - 2aq}{2}$.	2 points	
From the second arithmetic progression: $aq + 9 = \dfrac{a + aq^2}{2}$.	2 points	
By rearranging the above equations, the following simultaneous equations are obtained: $\left.\begin{aligned} aq^2 - 4aq &= 0 \\ aq^2 - 2aq + a &= 18 \end{aligned}\right\}$.	2 points	

Since $aq \neq 0$,	1 point	
from the first equation: $q = 4$.	1 point	
Therefore from the second equation: $a = 2$.	2 points	
Checking: geometric: 2; 8; 32; first arithmetic: 2; 8; 14; second arithmetic: 2; 17; 32.	2 points	
Therefore $a = 2$; $b = 8$; $c = 32$.	1 point	*This 1 point is also due if the terms of the geometric progression are only listed in the checking.*
Total:	**16 points**	

Solution 2.		
The terms of the first arithmetic progression are a; b; $c - a - 2b$.	1 point	
Thus $a + c - a - 2b = 2b$. (1)	2 points	
The terms of the second arithmetic progression are a; $b + 9$; c.	1 point	
Thus $a + c = 2b + 18$. (2)	2 points	
$b^2 = ac$. (3)	1 point	
From (1): $c = 4b$. (4)	1 point	
From (2) and (4): $a = 18 - 2b$. (5)	1 point	

From (3), (4) and (5): $b^2 = 4b\,(18 - 2b)$.	1 point	
Since $b > 0$,	1 point	
$b = 8$.	1 point	
$a = 2$.	1 point	
$c = 32$.	1 point	
Checking.	2 points	
Total:	**16 points**	

6.

a)

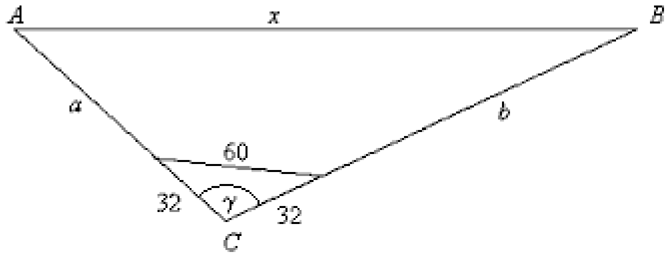
There are five choices for the first digit,	1 point	
and six choices for every other digit.	1 point	
$5 \cdot 6^5$ = 38 880 six-digit numbers can be formed.	1 point	*The correct answer is acceptable in either form.*
Total:	**3 points**	

b)

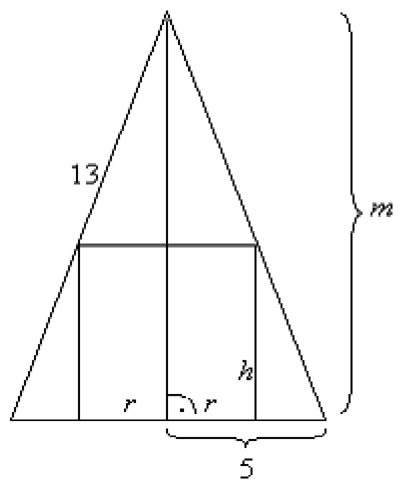
The six-digit number may end in zero or five.	1 point	*This point is also due if this statement is not formulated but the reasoning is correct.*
There are 5! that end in 0.	1 point	
There are $4 \cdot 4!$ that end in 5.	2 points	
Altogether 5! + $4 \cdot 4!$ = 216.	2 points	*The correct answer is acceptable in either form.*
Total:	**6 points**	

c)

The number of six-digit numbers with at least one repeating digit is obtained by subtracting from the number of all six-digit numbers made up of the given digits the number of six-digit numbers that consist of all different digits.	3 points	*These 3 points are also due if this reasoning is not written down in detail but it is clear that the solution uses this idea.*
The number of six-digit numbers without repetition is $5 \cdot 5!$	2 points	
The number of all cases is $5 \cdot 6^5$.		*The points for this result are awarded in question a).*
There is at least one repetition in $5 \cdot 6^5 - 5 \cdot 5!$ = 38 280 numbers.	2 points	*The answer is acceptable in either form.*
Total:	**7 points**	

7.		
a)		
	2 points	*2 points for the correct diagram that shows all relevant given information. These 2 points are also due if there is no diagram or it is incomplete but the correct solution shows the correct interpretation.*
Finding the angle γ : $\sin\frac{\gamma}{2}=\frac{30}{32}$.	1 point	
$\gamma \approx 139.27°$.	1 point	
The distances calculated from the speed of sound: $a=14\cdot 340=4760\ (\text{m})$ and	1 point	
$b=18\cdot 340=6120\ (\text{m})$.	1 point	
By applying the cosine rule to the triangle *ABC*: $x^2=4760^2+6120^2-2\cdot 4760\cdot 6120\cdot\cos 139.27°$.	2 points	
$x\approx 10\ 200$.	1 point	
The distance between the two sites is about 10 km.	1 point	*This point is awarded for rounding to the nearest kilometre.*
Total:	**10 points**	

b)		
Let the total distance covered be *s*. The walking time is $\frac{\frac{s}{2}}{2}+\frac{\frac{s}{2}}{5}$.	2 points	
The average speed: $\frac{s}{\frac{\frac{s}{2}}{2}+\frac{\frac{s}{2}}{5}}=\frac{2\cdot 2\cdot 5}{5+2}=$	2 points	
$=\frac{20}{7}\approx 2.86$.	1 point	
The average speed is ≈ 2.86 km/h.	1 point	
Total:	**6 points**	*Full credit should be given if the candidate does not express the total walking time but uses the fact that the average speed is the harmonic mean.*

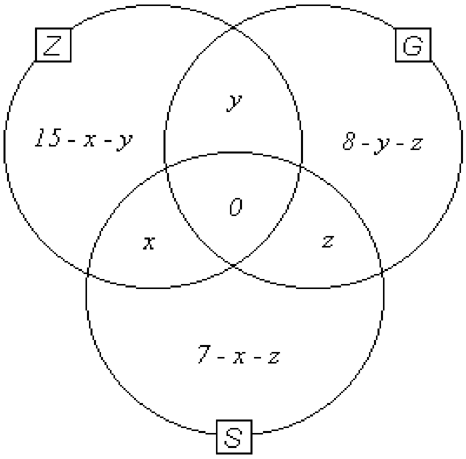
8.		
Solution 1.		
	2 points	*These 2 points are also due if there is no diagram but the correct solution shows the correct interpretation.*
$m=\sqrt{13^2-5^2}=12$ (cm).	1 point	
In right-angled triangles containing the same acute angle:	1 point	
$\frac{h}{5-r}=\frac{m}{5}$.	2 points	
Hence $h=\frac{m(5-r)}{5}=\frac{12\cdot(5-r)}{5}=12-2.4r$.	1 point	
The volume of the cylinder is $V(r)=r^2\pi(12-2.4r)=\pi(12r^2-2.4r^3)$, where $r\in\,]0;\,5[$.	2 points	*The 2 points are also due if it is not stated that* $r\in\,]0;\,5[$.
$V'(r)=\pi(24r-7.2r^2)$.	2 points	

A turning point may be where $24r - 7.2r^2 = 0$. $r \neq 0$, thus $r = \frac{10}{3}$.	3 points	*The 3 points are also due if there is no verbal explanation.*
For $r = \frac{10}{3}$ the derivative changes sign $+ \to -$, therefore $V(r)$ has a maximum.	1 point	
The radius of the cylinder is $\frac{10}{3}$ cm.	1 point	
Total:	**16 points**	

Solution 2.		
As far as finding *V*(*r*), this solution is the same as Solution 1.	9 points	
$V(r) = \pi(12r^2 - 2.4r^3) = 2.4\pi(5r^2 - r^3) =$ $= 2.4\pi \cdot r \cdot r \cdot (5 - r) = 9.6\pi \cdot \frac{r}{2} \cdot \frac{r}{2}(5 - r)$.	2 points	
It is enough to find the maximum of the product $\frac{r}{2} \cdot \frac{r}{2} \cdot (5 - r)$ where the sum $\frac{r}{2} + \frac{r}{2} + 5 - r$ is a constant, equal to 5.	2 points	
From the relationship of the arithmetic and geometric means, $\frac{r}{2} \cdot \frac{r}{2} \cdot (5 - r) \leq \left(\frac{\frac{r}{2} + \frac{r}{2} + 5 - r}{3}\right)^3 = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$.	1 point	
Equality only occurs if $\frac{r}{2} = 5 - r$, that is $r = \frac{10}{3}$.	1 point	
The radius of the cylinder is $\frac{10}{3}$ cm.	1 point	
Total:	**16 points**	

9.		
a)		
15 + 8 + 7 = 30, but there are only 18 students, thus there are 12 of them who learn to play two instruments.	3 points	
Total:	**3 points**	

b)



There are x students in the set $Z \cap S$. There are y students in the set $Z \cap G$.	1*point	*The 4 points are also due if these appear in the set diagram.*
No student learns to play both the saxophone and the guitar, thus there are $z = 0$ elements in the set $G \cap S$.	1 point	
$7 - x$ students play only the saxophone.	1 point	
$8 - y$ students play only the guitar.	1 point	
$x + y = 12$.	1 point	
$7 - x = 2 \cdot (8 - y)$.	1 point	
Solving the simultaneous equations $x = 5$; $y = 7$.	1 point	
Total:	**7 points**	

* This point is due for expressing this idea or for a verbal answer.

c)

There are $\binom{7}{2}$ ways to select two out of the 7 saxophonists.	1 point	
There are $\binom{8}{2}$ ways to select two out of the 8 guitarists.	1 point	
The number of favourable outcomes is $\binom{7}{2} + \binom{8}{2}$.	2 points	
There are $\binom{18}{2}$ ways to select two out of the 18 students.	1 point	
The probability in question is $\dfrac{\binom{7}{2}+\binom{8}{2}}{\binom{18}{2}} \approx 0.32$.	1 point	*The correct answer is acceptable in either form.*
Total:	**6 points**	